The Lorentz Force and Charge-to-Mass Ratio

Goals and Introduction

When an electrically-charged object moves in a magnetic field, it may experience a magnetic force if its velocity has a component that is perpendicular to the magnetic field direction. This force is called the *Lorentz force*. The magnitude of this force depends on the magnitude of the electric charge, the magnitude of the magnetic field, the magnitude of the velocity of the charged object, and the angle between the velocity and magnetic field (Eq. 1).

$$F_m = |q| vB \sin\theta \qquad \text{(Eq. 1)}$$

In the special case when the velocity is initially perpendicular to the magnetic field ($\theta = 90^\circ$), the Lorentz force will act as a centripetal force, and cause the object to move in a circular path. Recall that when an object is in circular motion, the magnitude of the centripetal force is related to how fast the object is moving, the radius of the circular path, and the mass of the object (Eq. 2).

$$F_c = \frac{mv^2}{r} \qquad \text{(Eq. 2)}$$

If we equate Eq. 1 and Eq. 2, for the case where the Lorentz force acts as the centripetal force, we can solve for the speed of the object in terms of the other quantities. Try deriving this result yourself!

$$v = \frac{|q| rB}{m} \qquad \text{(Eq. 3)}$$

If we are to perform an experiment to test this principle, we would need a method of creating moving, electrically-charged objects, or freeing electric charges from a material. By running an electric current through a material (a cathode) and heating it, it is possible to free electrons from the material. The electrons will be visible in this experiment because they will ionize an inert gas that exists in a closed tube. These electrons can then be accelerated by using an electric field, or potential difference. Recall that the magnitude of the change in electric potential energy of an electric charge that moves through a fixed potential difference, ΔV, depends only on the magnitude of the charge itself (Eq. 4). Note that the magnitude of the electric charge of an electron is equal to the fundamental electric charge, $e = 1.60 \times 10^{-19}$ C.

$$|\Delta PE| = |q\Delta V| = e|\Delta V| \qquad \text{(Eq. 4)}$$

Assuming that the electrons were just freed from the material, they likely don't have very much kinetic energy. This means that the change in electric potential energy that an electron experiences, moving through a potential difference, will result in the electron gaining kinetic energy. This means we can equate Eq. 4 to the definition of kinetic energy and solve for the final speed of the electrons, *v*.

$$v = \sqrt{\frac{2e|\Delta V|}{m}} \qquad \text{(Eq. 5)}$$

It would be very difficult to use a device to directly measure the speed of the electrons that were accelerated by this potential difference. But, if these electrons then move into a region where there is a magnetic field that is perpendicular to the velocity of the electrons, they will move in circular motion due to the Lorentz force. This means that we could now equate Eq. 3 and Eq. 5, again, using the fact that the magnitude of the electric charge of the electron is equal to the fundamental electric charge, *e*.

$$\frac{erB}{m} = \sqrt{\frac{2e|\Delta V|}{m}} \qquad \text{(Eq. 6)}$$

By rearranging this equation, we can show solve for the charge-to-mass ratio of the electron (Eq. 7). Try deriving this result yourself!

$$\frac{e}{m} = \frac{2|\Delta V|}{r^2 B^2} \qquad \text{(Eq. 7)}$$

This result means that if we know the potential difference through which the electrons are accelerated, the radius of the circular path of the electrons, and the magnitude of the magnetic field.

In today's lab, you will use a Helmholtz pair-coil apparatus to generate and control the magnetic field. The derivation of this field is beyond the scope of this document, and perhaps the course you are in. The magnitude of the magnetic field depends on the number of turns of wire in the coils, *N*, the current through the coils, *I*, and the radius of the coils, *R*, as given in Eq. 8.

$$B = \frac{8\mu_0 NI}{R\sqrt{125}} \qquad \text{(Eq. 8)}$$

With the ability to determine the magnetic field, we will be able to measure the charge-to-mass ratio of the electrons that are freed from the cathode and accelerated through the potential difference. We can then compare our results to a predicted ratio, found using the accepted values for the magnitude of electric charge and mass of an electron.

Be sure to read all of the warnings, notes and procedure carefully! The equipment you will use today is very sensitive and expensive. It needs to be preserved for future classes. If you are unsure about anything, ask your TA before proceeding.

Goals: (1) Determine the charge-to-mass ratio of an electron

(2) Develop a better understanding of the Lorentz force and how it affects moving electrically-charged objects

Procedure

Equipment – Helmholtz pair-coil apparatus with electron tube, 0 – 30 V DC 1 A wall power source, 5 V DC fixed potential power source, 0 – 30 V DC 5 A wall power source, 50 V DC fixed potential power source, two ammeters, switch, compass, ruler

NOTE: There is a very specific order in which the various power supplies must be turned on and turned off. PAY ATTENTION to the instructions, and if you are unsure about anything, check with your TA.

1) The equipment should actually already be hooked up for you, but examine the connections and be sure that they are as shown below in Figure 1. Read the labels on the pair-coil apparatus carefully so you know where all connections are made. In verifying, realize that the ground of the power supplies are all connected to a common ground. Be sure that all of the knobs on the power supplies are turned all the way counter-clockwise and that all switches are off. The physical switch on the table should be in the vertical position so the connection is not closed.

2) Use the compass to ensure that there are no strong, external magnetic fields near the electron tube in the pair-coil apparatus. It is likely that the apparatus has already been situated where it needs to be, so don't move it unless absolutely necessary, and check with your TA before doing so.

3) Examine the pair-coil apparatus and **record** the number of turns, *N*, for the coils.

4) Use the ruler to **measure and record** the inner and outer diameter of one of the coils. Verify that the other has similar values.

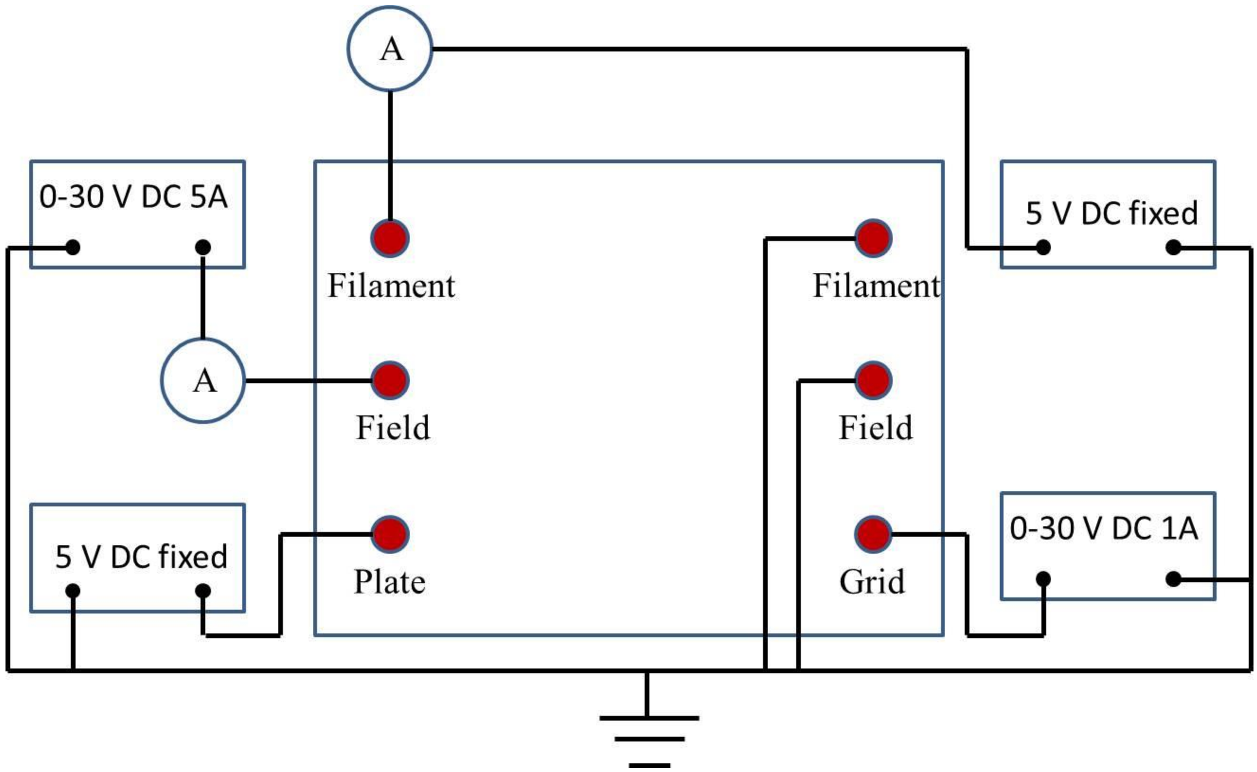

Figure 1

5) Flip the switch at the wall for the *filament* (the 5 V DC fixed power supply), to begin to heat the filament. Wait at least 2 minutes before proceeding to the next step!

6) Turn on the *plate* voltage by turning on the 50 V DC fixed power supply. Move the switch on the table from the vertical position to the horizontal position so that it is closed. **Record** the voltage as it reads on the wall meter. This is the potential difference, ΔV, that is used to accelerate the electrons.

7) Turn on the *grid* by turning on the 0-30 V DC 1 A power supply. Adjust the knob on the power supply to get the beam to be collimated. It should look like a regular cylindrical beam with a diameter of about 2 mm. See Figure 2 for the desired outcome for the beam.

WARNING: In the next part of the experiment, you will start to bend the electron beam. DO NOT bend the beam all the way to the inner circle on the plate within the tube.

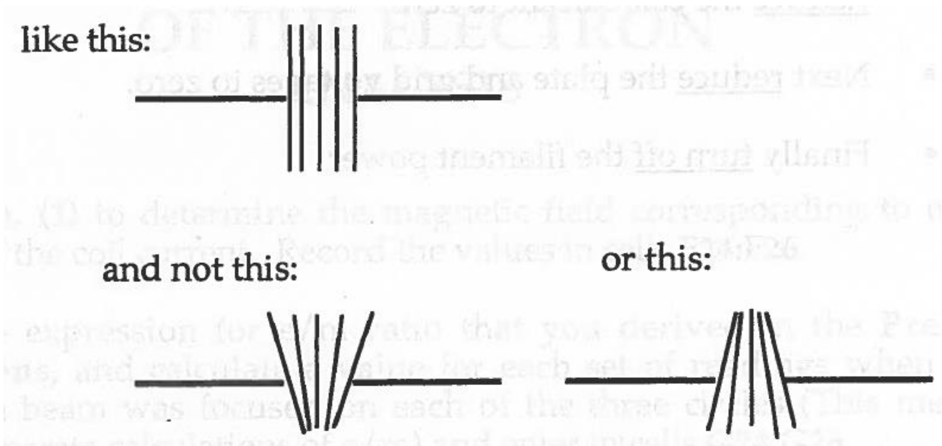

Figure 2

8) Turn on the *field* by turning on the 0-30 V DC 5 A power supply. Observe that as you turn the knob on the power supply counter-clockwise, the electron beam bends. Note the reading on the ammeter of the current in the coils as you turn the knob, here. That value should NEVER exceed 4.7 A in this experiment. Don’t let it happen.

9) **Record** the radii of the circles on the plate within the electron tube, which are 0.5 cm, 1.0 cm, 1.5 cm, and 2.0 cm. The circles are coated with an electron-reactive substance that will appear to glow when the beam strikes the circle.

10) Turn the knob on the 0-30 V DC 5 A power supply to bend the beam until it hits the circle with the radius of 2.0 cm, on the outer edge of the plate within the electron tube. **Record** the radius of the circle you are striking and the current in the coils.

11) Turn the knob on the 0-30 V DC 5 A power supply to bend the beam until it hits the circle with the radius of 1.5 cm, the first circle in from the outer edge of the plate within the electron tube. **Record** the radius of the circle you are striking and the current in the coils.

12) Turn the knob on the 0-30 V DC 5 A power supply to bend the beam until it hits the circle with the radius of 1.0 cm, the second circle in from the outer edge of the plate within the electron tube. **Record** the radius of the circle you are striking and the current in the coils.

13) The equipment must be turned off in the reverse order it was turned on. So,

- Turn the knob on the 0-30 V DC 5 A power supply all the way counter-clockwise, and then turn off that power supply. This turns off the *field*.

- Turn the knob on the 0-30 V DC 1 A power supply all the way counter-clockwise, and then turn off that power supply. This turns off the *grid*.

- Move the switch on the table to the vertical position. This turns off the *plate*.

- Flip the switch on the wall for the 5 V DC fixed power supply. This turns off the *filament*.

As always, be sure to organize your data records for presentation in your lab report, using tables and labels where appropriate.

Data Analysis

Use the inner and outer diameters of the coils you measured to calculate an average diameter of the coils. Then, calculate an average radius of the coils, R.

Use the average radius of the coils, the number of turns of wire in the coils, and the current that you recorded in steps 10, 11, and 12 to calculate the magnitude of the magnetic field for each case of circular motion of the electron beam.

Consider that when the beam was bent to hit a circle on the plate within the electron tube, it was moving in a circle with a diameter equal to the radius of the circle that you recorded on the plate. We need to find the actual radius of the circular motion. Use the radii of the circles on the plate you recorded in steps 10, 11, and 12 to calculate the radii of the circular motion of the electron beam in each case.

Question 1: How do we arrive at Eq. 3? Explain your answer and show your derivation of Eq. 3.

Question 2: How do we arrive at Eq. 7? Explain your answer and show your derivation of Eq. 7.

Use Eq. 7 to calculate the charge-to-mass ratio for an electron using the plate potential difference, and the radius of the circular motion, and the magnitude of the magnetic field for each of the three cases. Then, calculate the mean value you found for the charge-to-mass ratio.

Look up the accepted values for the magnitude of the electric charge and mass of an electron and calculate the predicted value of the charge-to-mass ratio.

Error Analysis

Calculate the percent error between the experimental value and predicted value of the charge-to-mass ratio.

$$\%error = \frac{\left| e/m_{\text{predict}} - e/m_{\text{exp}} \right|}{e/m_{\text{predict}}} \times 100\%$$

Question 3: How well did the experiment match the predicted value? Can we conclude that this beam really consisted of electrons? Explain why or why not.

Questions and Conclusions

Be sure to address Questions 1 through 3 and describe what has been verified and tested by this experiment. What are the likely sources of error? Where might the physics principles investigated in this lab manifest in everyday life, or in a job setting?

Pre-Lab Questions

Please read through all the instructions for this experiment to acquaint yourself with the experimental setup and procedures, and develop any questions you may want to discuss with your lab partner or TA before you begin. Then answer the following questions and type your answers into the Canvas quiz tool for "The Lorentz Force and Charge-to-Mass Ratio," and submit it before the start of your lab section on the day this experiment is to be run.

PL-1) Marwin is performing the Lorentz force lab experiment and he has found that the pair-coil has 120 turns, has a radius of 0.15 m, and is carrying a current of 1.5 A. What is the magnitude of the magnetic field in the pair-coil apparatus? Express your answer in milliteslas, mT, and use three significant figures.

PL-2) Eliza is performing the Lorentz force lab experiment and she is deflecting the beam to the circle on the plate with a radius of 2.0 cm. What is the radius of the circular path of the electron beam? Express your answer in meters, m.

PL-3) Eliza and Marwin are performing the Lorentz force lab experiment and they found that the pair-coil has 120 turns, has a radius of 0.15 m, and is carrying a current of 1.5 A. The plate voltage was 50.0 V. If the beam is currently deflected to the circle on the plate with a radius of 2.0 cm, what is the charge-to-mass ratio that they calculate?

A) 9.11×10^{31} C/kg

B) 1.60×10^{-19} C/kg

C) 1.16×10^{-12} C/kg

D) 8.59×10^{11} C/kg

PL-4) During the experiment, Eliza should be sure that the current through the *field* connections never exceeds what value? Express your answer in amps, A.

PL-5) Suppose that the electron beam was bent to strike the outer circle on the plate within the electron tube by the magnetic field. If the electrons were slowed down (due to a decreased plate potential difference), how would the radius of the circular motion be affected? Consider Eq. 3 in choosing your response.

A) The radius would stay the same because even though the electrons will move more slowly, the magnetic force will be weaker.

B) The radius would decrease.

C) The radius would increase.

D) The electrons wouldn’t be able to enter the tube because the gravitational force would be too strong compared to the electrical force.